International Journal of Applied Mathematics
& Statistical Sciences (IJAMSS)
ISSN(P): 2319-3972; ISSN(E): 2319-3980
Vol. 3, Issue 1, Jan 2014, 1-4

International Academy of Science, Engineering and Technology
Connecting Researchers; Nurturing Innovations

MATHEMATICAL MODEL OF COMPARTMENTAL EPIDEMIC

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ABSTRACT

In this paper, we study a model of compartmental epidemic in which the immunity for the recovery class is zero and they move back into the susceptible class. We observed that there are two equilibria points and both equilibria points are under control.

KEYWORDS: Compartmental Epidemic, Equilibria Point, Immunity

1. INTRODUCTION

There are many studies on HIV/AIDS which includes Diekmann et al (2009), Ayeni et al (2010), Oluyo (2013). All these researchers studied different ways of controlling the spread of this epidemic disease. Of a particular interest is Diekmann (2009), who constructed next matrices for compartmental epidemic models on which our own work is based. We revisited his work and considered a case when the recovery class does not have immunity and they were not removed from the system. We observed that the two equilibria points are stable unconditionally and the per capital birth and death rate (μ) has effect on the population but no effect on the stability of the equilibrium point.

2. THE MODEL

We modified Diekmann O et al (2009) model by considering a case when the recovery class does not have immunity and they were not removed from the system.

Arising from the above, the relevant mathematical equations are:

$$\frac{dS}{dt} = \mu N - \frac{\beta SI}{N} - \mu S + \mu R$$

$$\frac{dE_1}{dt} = \frac{p\beta SI}{N} - (v_1 + \mu)E_1 + p\mu R$$

$$\frac{dE_2}{dt} = (1-p)\frac{\beta SI}{N} - (v_2 + \mu)E_2 + (1-p)\mu R \tag{1.1}$$

$$\frac{dI}{dt} = v_1 E_1 + v_2 E_2 - (\gamma + \mu)I + (\mathrm{v}_1 + \mathrm{v}_2)R$$

$$\frac{dR}{dt} = \gamma I - \mu R$$

Where S susceptible, E_1 latently infected category 1, E_2 latently infected category 2, I infectious, R recovery/ not removed /no immune, β transmission rate, μ the per capital birth and death rate, p ratio of E_1 in the population, (1-p) ratio of E_2 in the population, v_1 rate of leaving E_1, v_2 rate of leaving E_2, γ rate of leaving the infectious state, $N=S+E_1+E_2+I+R$

3. THE CRITICAL POINTS

There exits the following two equilibria correspond to the system (1)

- E_0 (185,0,0,0,0)
- E_1 (-2143.141172, 208.8163656, 0,-0.2105743539,-110.4971052)

Stability Criteria

Theorem 1: The equilibria E_0 and E_1 are asymptotically stable unconditionally.

Proof: The general variation matrix M corresponding to system (1.1) is

$$M=\begin{pmatrix} -\mu & 0 & 0 & 0 & \mu \\ 0 & -(\mathrm{v}_1+\mu) & 0 & 0 & p\mu \\ 0 & 0 & -(\mathrm{v}_2+\mu) & 0 & (1-\mathrm{p})\mu \\ 0 & v_1 & v_2 & -(\gamma+\mu) & (\mathrm{v}_1+\mathrm{v}_2) \\ 0 & 0 & 0 & \gamma & -\mu \end{pmatrix}$$

At equilibrium point E_0 (185,0,0,0,0) the variation matrix M_o is given by

$$M_o=\begin{pmatrix} -\mu & 0 & 0 & 0 & \mu \\ 0 & -(\mathrm{v}_1+\mu) & 0 & 0 & p\mu \\ 0 & 0 & -(\mathrm{v}_2+\mu) & 0 & (1-\mathrm{p})\mu \\ 0 & v_1 & v_2 & -(\gamma+\mu) & (\mathrm{v}_1+\mathrm{v}_2) \\ 0 & 0 & 0 & \gamma & -\mu \end{pmatrix}$$

And the corresponding eigenvalues to the matrix above are λ_1=-0.1, λ_2=-0.11, λ_3=-0.076799353, λ_4=-0.1266010032+0.018007705521I, λ_5=-0.1266010032-0.018007705521I, which shows that the disease free equilibrium point is asymptotically stable.

At equilibrium point E_1 (-2143.141172, 208.8163656, 0,-0.2105743539,-110.4971052) the variation matrix M_1 is given by

$$M_1=\begin{pmatrix} 0.2105743539\dfrac{\beta}{N}-\mu & 0 & 0 & 2143.141172\dfrac{\beta}{N} & \mu \\ -0.2105743539\dfrac{p\beta}{N} & -(\mathrm{v}_1+\mu) & 0 & -2143.141172\dfrac{p\beta}{N} & p\mu \\ 0 & 0 & -(\mathrm{v}_2+\mu) & 0 & (1-\mathrm{p})\mu \\ 0 & 0 & v_2 & -(\gamma+\mu) & (\mathrm{v}_1+\mathrm{v}_2) \\ 0 & 0 & 0 & \gamma & -\mu \end{pmatrix}$$

And the corresponding eigenvalues to the matrix above is λ_1=-0.099965280, λ_2=-0.08763932023, λ_3=-0.11, λ_4=-0.11, λ_5=-0.1323606798, which shows that the endemic equilibrium point is asymptotically stable.

Simulations: The system (1.1) is solved numerically. Since infection takes into account the births and death of human.

In the numerical solution the initial value used are S(0)=100, E_1(0)=30,E_2(0)=40, I(t)=10, R(t)=5 and the

following data were used β=0.03, μ=0.1, γ=0.02, p=1, v_1=0.01.v_2=0.01 and the per capital birth and death rate has effect on the population of Susceptible and infectious class but no effect on the stability of the equilibria points.

4. RESULTS

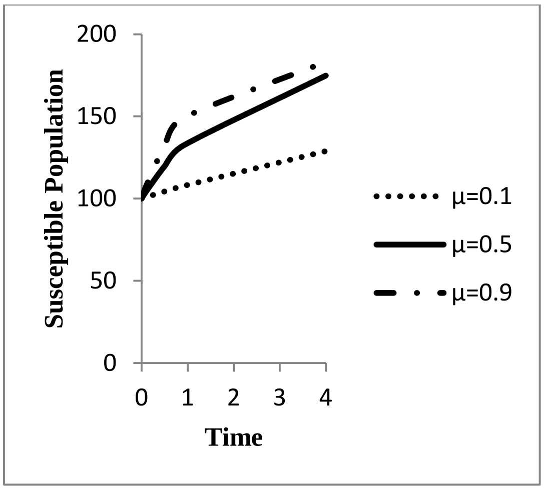

Figure 1: Susceptible Population (W) of E_0 with Time for Various Values of M

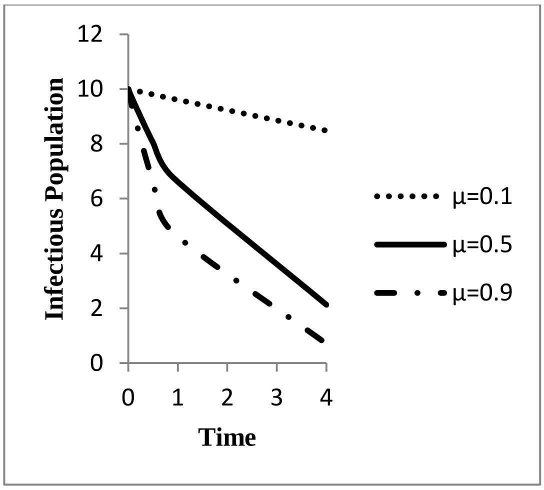

Figure 2: Infectious Population (Z) of E_0 with Time at Various Values of M

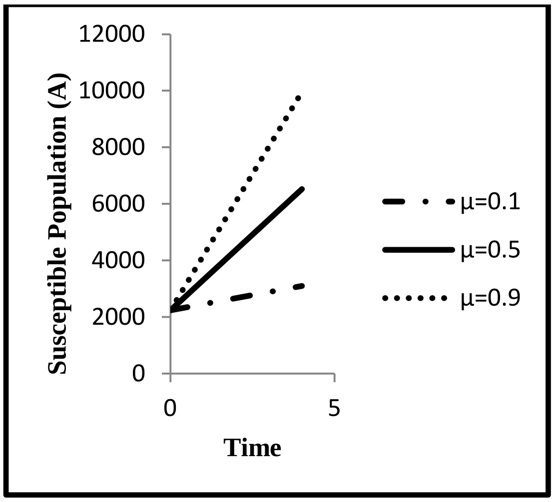

Figure 3: Susceptible Population (A) of E_1 with Time for Various

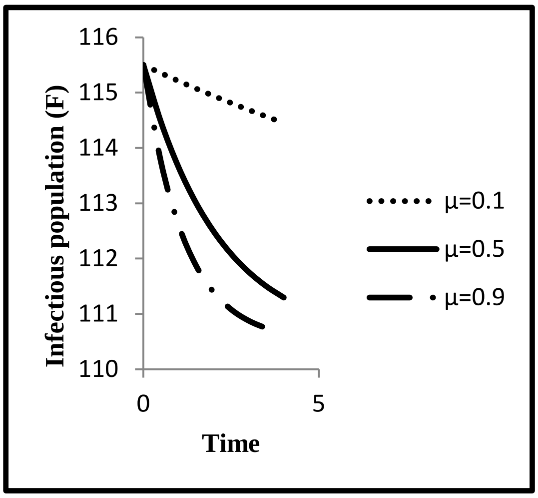

Figure 4: Infectious Population (F) of E_1 with Time at Various Values of M

5. DISCUSSIONS OF RESULTS

From the solution obtained numerically, we observed that increment in per capital birth and death rate (μ) populates the susceptible class and depopulates the infectious class and also we observed that it has no effect on the stability of the two equilibria points.

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